

1. Research Methodology

- a. What is meant by the supply chain (SC) coordination problem and does it apply to all types of SC's? Does the Bullwhip effect relate to all types of SC's? Also does it relate to SC coordination?
- b. Define what is meant by SC incentive contracts. In what situations do incentive contracts play a role? Describe the principal kinds of incentive contracts. Describe two research results on the performance of incentive contracts.
- c. Explain the basic Newsvendor model. Why is it useful for SC models?
- d. Describe Shapley Value and explain how it may be of use in SC coordination. What is meant by the core of a cooperative N-person game? What properties of a game solution does the Shapley Value solution provide?
- e. What additional research do you believe will be helpful in this area?

2. Stochastic Models

Part A. The Newsboy Problem.

Consider the famous Newsboy Problem or the Single Period Inventory problem.

Let D be the demand for that one period with *pdf* $f(x)$ and *cdf* $F(x)$, and Q be the order quantity (the decision variable). For convenience, let's assume D to be a continuous random variable. Also define the followings:

C_o = the overage cost (cost per unit of positive inventory remaining at the end of the period).

C_u = the underage cost (cost per unit of unsatisfied demand or cost per unit of negative ending Inventory).

$[Q-D]^+ = \max \{Q-D, 0\} = Q - D$ if $(D \leq Q)$ and 0 otherwise.

$[Q-D]^- = \max \{D-Q, 0\} = D - Q$ if $(D \geq Q)$ and 0 otherwise.

Notice that $[Q-D]^+$ and $[Q-D]^-$ are the positive and negative remaining inventory at the end of the period, respectively.

Now we can define the expected value of the total cost (TC) to be

$$E [TC(Q)] = C_o E[[Q-D]^+] + C_u E[[Q-D]^-] \quad (1)$$

Question 1: Derive the mathematical expression for $E [TC(Q)]$ and find the optimal order quantity Q^ . Show the necessary and sufficient conditions for Q^* .*

Part B. Re-interpretation of the meaning C_o and C_u using the standard inventory model parameters.

Define

S = Selling price per unit

c = Variable cost per unit

h = holding cost per unit of inventory remaining in stock at the end of the period.

p = shortage cost per unit charge against the number of back orders at the end of the period.

$\mu = E(D)$ the expected value of the demand per period.

Now we can define a more general cost function for one period as:

$$\begin{aligned} TC(Q) &= \text{purchase cost} + \text{holding cost} + \text{shortage cost} - \text{Revenue} \\ &= cQ + h[Q - D]^+ + p[Q - D]^- - S \min \{Q, D\} \end{aligned} \quad (2)$$

Question 2: Derive the mathematical expression for $E[TC(Q)]$ of (2) and find the optimal order quantity Q^* in terms of h, p, c, S . Show the necessary and sufficient conditions for Q^* .

Part C. Extension to back-order infinite horizon problem (multi-period problem)

Let D_1, D_2, D_3, \dots be the infinite sequence of demands in periods 1, 2, 3, Assume D_1, D_2, D_3, \dots to be iid random variables with pdf $f(x)$ and cdf $F(x)$. Notice that

$$\text{Number of units sold in period 1} = \min\{Q, D_1\},$$

$$\text{Number of units sold in period 2} = \max\{D_1 - Q, 0\} + \min\{Q, D_2\},$$

(units back orders + units sold in period 2)

$$\text{Number of units sold in period 3} = \max\{D_2 - Q, 0\} + \min\{Q, D_3\}, \text{ etc.}$$

Question 3: Let $TC_n(Q)$ be the total cost over n periods. Using the cost parameters in part B, show that

$$E[TC_n(Q)] = cQ + (c - S)E(D_1 + D_2 + \dots + D_{n-1}) - S \cdot E\{\min(Q, D_n)\} + nL(Q)$$

where

$$L(Q) = h \int_0^Q (Q - x)f(x)dx + p \int_Q^\infty (x - Q)f(x)dx.$$

Question 4: Show that the average cost for infinite period problem,

$$E[TC(Q)] = \lim_{n \rightarrow \infty} \frac{TC_n(Q)}{n} = (c - S)\mu + L(Q) \quad (3)$$

And hence the optimal order quantity is given by

$$F(Q) = \frac{p}{p + h} \quad (4)$$

It is interesting to note that Q^* only depends on p and h .

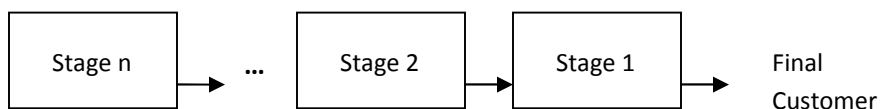
The right hand side of (4) is known as the *CRITICAL RATIO*: $CR = \frac{p}{p + h}$

3. Supply Chain Management

The question consists of two parts – answer both parts; part 1 is weighted 60%; part 2 is weighted 40%.

Part 1: Recent research by Guiffrida et al. (2007), Garg et al. (2006) and Guiffrida and Nagi (2006) all model delivery time performance of a single product to the final customer in a serial supply chain. A common limitation found in the model formulations presented in each of these papers is the assumption of a make-to-order orientation within the serial supply chain. Clearly, not all serial supply chains operate in a make-to-order orientation. Hence, a generalized model for evaluating delivery time performance to the final customer in a serial supply chain should accommodate both make-to-order and make-to-stock orientations.

Consider the following serial supply chain:



Let the total delivery time to the final customer (W) be defined as $W = \sum_{i=1}^N X_i$ where X_i is the processing time for the i^{th} stage of an N stage serial supply chain. In the make-to-order orientation found in the above cited literature the final customer places a demand on Stage 1 of the chain which in turn places a demand on Stage 2 of the chain, ..., which in turn ultimately places a demand on Stage N of the chain.

In a make-to-stock orientation, inventory held at an upstream stage of the supply chain may exist and thus not all stages of the supply chain are required to satisfy the customer order. Always starting with Stage 1, the number of stages required to complete the customer order (and hence the total delivery time) is a random variable where $P(N = n) = p_n$ for $n = 1, 2, \dots$. By defining W as a random sum of random variables the delivery performance models found in Guiffrida et al. (2007), Garg et al. (2006) and Guiffrida and Nagi (2006) can be generalized to accommodate both make-to-order and make-to-stock orientations.

Derive the expected value and variance of total delivery time W when $P(N = n) = p_n$ for $n = 1, 2, \dots$ (Assume independence among the X_i and assume that there is no waiting time between stages).

Part 2: Let $f_w(w)$ define the probability density function for W . Discuss how you would determine an approximate form for $f_w(w)$ under the conditions described in Part 1.

4. Simulation

Variance reduction techniques have been called the “free lunch,” as they allow us to get more efficient solutions with the same effort. The questions below all have to do with variance reduction, and rely on the following situation: Ships arrive at a harbor with interarrival times that are IID exponential random variables. The harbor has a dock with two berths and two cranes for unloading the ships; ships arriving when both berths are occupied join a FIFO queue. The time for one crane to unload a ship is distributed uniformly between 1 and 2 days. If only one ship is in the harbor, both cranes unload the ship and the (remaining) unloading time is cut in half. When two ships are in the harbor, one crane works on each ship. If both cranes are unloading one ship when a second ship arrives, one of the cranes immediately begins serving the second ship, and the remaining service time of the first ship is doubled. Assume that no ships are in the harbor at time 0. We are interested in computing the minimum, maximum, and average time the ships are in the harbor.

- a. Consider using antithetic variates (AV) for the above problem. Specifically, which input random variables should be generated antithetically, and how could proper synchronization be maintained?
- b. Suppose that thought is being given to replacing the two existing cranes with two faster ones. Specifically, single-crane unloading times for a ship would be distributed uniformly between 0.5 and 1 day; everything else remains the same. Discuss the proper application and implementation of common random numbers (CRN) to compare the original system to the proposed system.
- c. Now assume that in the proposed system (part b above), the single-crane unloading times follow a normal distribution rather than a uniform distribution. Further, we use the *Polar* method to generate these normal random variates. Discuss the effect of changing the unloading time distribution from uniform to normal on the application of CRN when comparing this system to the original system (part a). Hint: The Polar method requires a random number of $UN(0,1)$ numbers to generate a pair of normal random variates.
- d. Briefly, comment on the pitfalls, if any, of applying both AV and CRN simultaneously, i.e., AV within a system, and CRN across systems, for the same simulation experiment.

Be specific and brief in your answers to the above questions.